Northgate written calculations policy

Rationale

This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the new 2014/15 National Curriculum. The policy aims to link key manipulatives and representations in order that the children can be vertically accelerated through each strand of calculation. This policy is to ensure consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching, allowing for deeper conceptual understanding and fluency. As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However, it is expected that the majority of children in each class will be working at age-appropriate levels as set out.

The importance of mental mathematics

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

To add and subtract successfully, children should be able to:

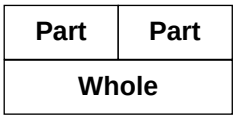
- recall all addition pairs to 9 + 9 and number bonds to 10
- recognise addition and subtraction as inverse operations
- add mentally a series of one digit numbers (e.g. 5 + 8 + 4)
- add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. 600 + 700, 160 — 70)
- partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)
- use estimation by rounding to check answers are reasonable

To multiply and divide successfully, children should be able to:

- add and subtract accurately and efficiently
- recall multiplication facts to 12 x 12 = 144 and division facts to 144 ÷ 12 = 12
- use multiplication and division facts to estimate how many times one number divides into another etc.
- know the outcome of multiplying by 0 and by 1 and of dividing by 1
- understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
- recognise factor pairs of numbers (e.g. that 15 = 3 x 5, or that 40 = 10 x 4) and increasingly able to recognise common factors
- derive other results from multiplication and division facts and multiplication and division by 10 or 100 (and later 1000)
- notice and recall with increasing fluency inverse facts
- partition numbers into 100s, 10s and 1s or multiple groupings
- understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
- understand the effects of scaling by whole numbers and decimal numbers or fractions
- understand correspondence where n objects are related to m objects
- investigate and learn rules for divisibility

Progression in addition and subtraction

Addition and subtraction are connected.



Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

Gradation of difficulty- addition:	**Gradation of difficulty- subtraction:**
1. No exchange.	1. No exchange.
2. Extra digit in the answer.	2. Fewer digits in the answer.
3. Exchanging ones to tens.	3. Exchanging tens for ones.
4. Exchanging tens to hundreds.	4. Exchanging hundreds for tens.
5. Exchanging ones to tens and tens to hundreds.	5. Exchanging hundreds to tens and tens to ones.
6. More than two numbers in calculation.	6. As 5 but with different number of digits.
7. As 6 but with different number of digits.	7. Decimals up to 2 decimal places (same number of decimal places).
8. Decimals up to 2 decimal places (same number of decimal places).	8. Subtract two or more decimals with a range of decimal places.
9. Add two or more decimals with a range of decimal places.	

Key Vocabulary	
Addition	**Subtraction**
Add, addition, total, plus, more than, and, altogether, increase, equals, make, sum Consistent use of the vocabulary throughout maths sessions, in conjunction with visual models and practical experiences is essential to developing the concept of addition.	Subtract, subtraction, take away, minus, less than, difference, decrease, leave, how many left? Consistent use of the vocabulary throughout maths sessions, in conjunction with visual models and practical experiences is essential to developing the concept of subtraction.

Foundation

In the foundation stage calculation is taught using practical activities that are

- Visual
- Fun
- Meaningful
- Active

In Foundation we assess against the Development Matters Statements.

(22-36 months)

- Knows that a group of things changes in quantity when something is added or taken away.

(30-50 months)

- Shows an interest in number problems.

(40-60 Months)

- Finds the total number of items in two groups by counting all of them.
- Finds one more or one less from a group of up to five objects, then ten objects.
- In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting.

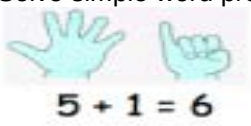
By the end of the Foundation Stage, pupils are assessed against the following Early Learning Goals:

- To count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less that a given number.
- To use quantities and objects, add and subtract two single-digit numbers and count on or back to find the answer.

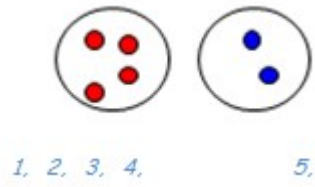
<table>
<tr><th colspan="2">Pupils will begin to record in the context of play or practical activities and problems.</th></tr>
<tr><th>Addition</th><th>Subtraction</th></tr>
<tr><td>

Begin to relate addition to combining two groups of objects

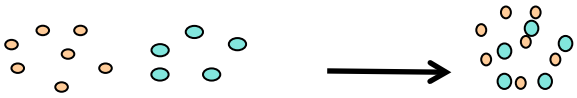
Make a record in pictures, words or symbols of addition activities carried out.
Construct number sentences to go with practical activities.
Use of games, songs and practical activities to begin using vocabulary.
Solve simple word problems using their fingers



Write number sentences in context including the number of children present for the day and the number that are absent, the number of children having school dinners and packed lunches etc.
Add two single digit numbers using objects.



Putting together – two or more amounts or numbers are put together to make a total.
7 + 5 = 12

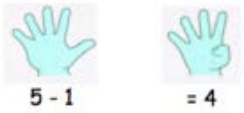


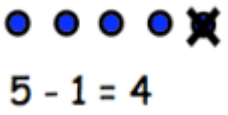
Count one set, then the other set. Combine the sets and count again. Starting at 1.

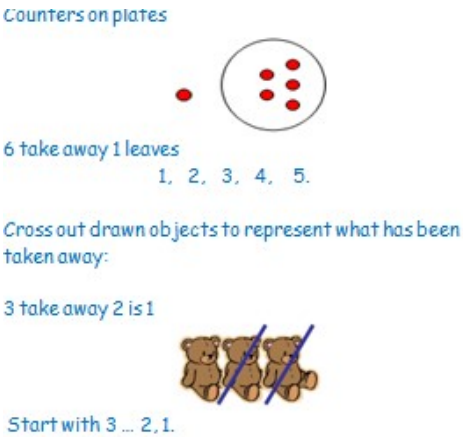
</td><td>

Begin to relate subtraction to 'taking away'

Make a record in pictures, words or symbols of subtraction activities already carried out.
Use of games, songs and practical activities to begin using vocabulary.
Construct number sentences to go with practical activities .
Relate subtraction to taking away and counting how many objects are left.
Subtract two single digit numbers







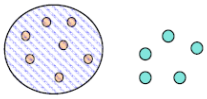
</td></tr>
</table>

Combining two sets

This stage is essential in children beginning to calculate rather than count.

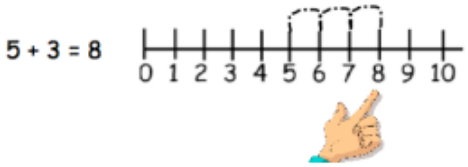
Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.

Counters:



Start with 7, then count on 8, 9, 10, 11, 12

For those pupils that are ready we would progress onto a number square or a number line to count on and back. We encourage them to start at the largest number first.

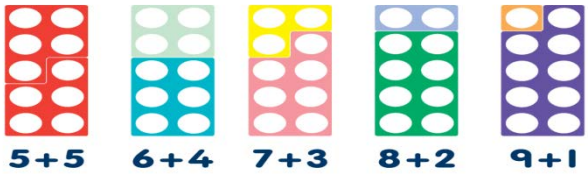


Numicon:

Make a set of 5 add 3 = 8.

5 + 3 = 8

Numicon is used as a way of counting and to write addition sentences.



Practically, for example:

Group objects on a table then cover some to visualize the calculation:

2 less than 4 is 2

Start with 2... 3, 4.

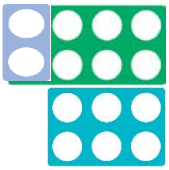
Coins

I had 10 pennies. I spent 4 pence. How much do I have left? Start with 10... 9, 8, 7, 6.

Counting backwards along a number line using finger

Numicon:

8 – 2 = 6



Year 1

Represent and use number bonds and related subtraction facts within 20.
Add and subtract one-digit and two-digit numbers to 20, including zero.
Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = [] - 9.
Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.

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<tr><td>
<u>Combining two sets</u>
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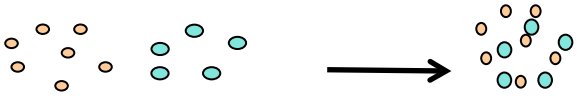
Count one set, then the other set. Combine the sets and count again. Starting at 1.
Counting along the bead bar, count out the 2 sets, then draw them together, count again. Starting at 1.

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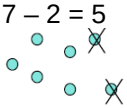
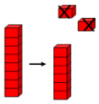
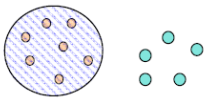
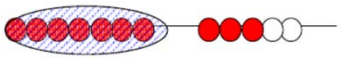
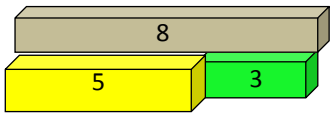
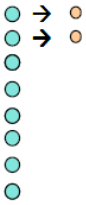
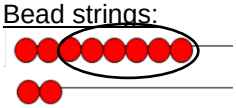
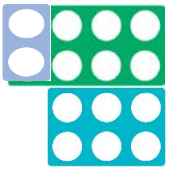
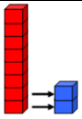
<u>Taking away</u>
Where one quantity is taken away from another to calculate what is left.
7 – 2 = 5

Multilink towers - to physically take away objects.

</td></tr>
<tr><td>
<u>Combining two sets</u>
This stage is essential in children beginning to calculate rather than count.
Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.
<u>Counters:</u>

Start with 7, then count on 8, 9, 10, 11, 12
<u>Bead strings:</u>

<u>Numicon:</u>
Make a set of 5 add 3 = 8.
5 + 3 = 8

<u>Multilink Towers:</u>

<u>Cuisenaire Rods, Dienes:</u>


<u>Number tracks:</u>


</td><td>
<u>Finding the difference</u>
Two quantities are compared to find the difference.
8 – 2 = 6
<u>Counters:</u>

<u>Bead strings:</u>

Make a set of 8 and a set of 2. Then count the gap.
<u>Numicon:</u>
8 – 2 = 6

<u>Multilink Towers:</u>

Cuisenaire Rods, Dienes and arrow cards.
<u>Number tracks:</u>
Start with the smaller number and count the gap to the larger number.
1 set within another (part-whole model)
The quantity in the whole set and one part are known, and may be used to find out how many are in the unknown part. Using a range of practical resources.
8 – □ = 6
</td></tr>
</table>

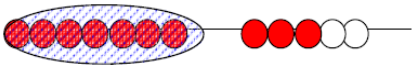
Bridging through 10s

Before bridging, children need to be confident with all the number facts up to ten and add a one digit number to a ten.

10 + 3 = 13

This stage encourages children to become more efficient and begin to employ known facts.

Bead string:



7 + 5 is decomposed / partitioned into 7 + 3 + 2.
The bead string illustrates 'how many more to the next multiple of 10?' (children should identify how their number bonds are being applied) and then 'if we have used 3 of the 5 to get to 10, how many more do we need to add on?' (ability to decompose/partition all numbers applied).

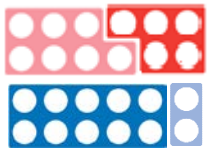
Number track:

Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

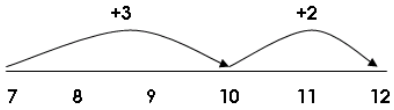
Numicon:

Make a set of 7 and a set of 5. Then count on from 7.

7 + 5 = 12



To be used alongside a number line:



Begin counting in ones before partitioning digits.

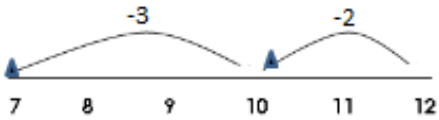
Bead string:

12 – 7 is decomposed / partitioned in 12 – 2 – 5.
The bead string illustrates 'from 12 how many to the last/previous multiple of 10?' and then 'if we have used 2 of the 7 we need to subtract, how many more do we need to count back?' (ability to decompose/partition all numbers applied).

Number Track:

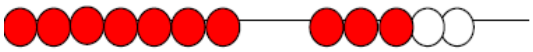
Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number Line:



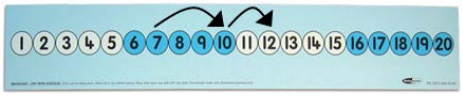
Counting up or 'Shop keepers' method.

Bead string:

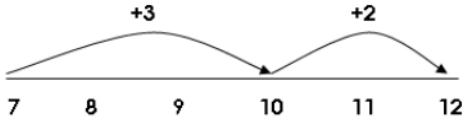


12 – 7 becomes 7 + 3 + 2.
Starting from 7 on the bead string 'how many more to the next multiple of 10?' (children should recognise how their number bonds are being applied), 'how many more to get to 12?'.

Number Track:



Number Line:



Notes and guidance

Pupils combine and increase numbers, counting forwards and backwards. They discuss and solve problems in familiar practical contexts, including using quantities. Problems should include the terms put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

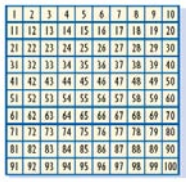
Year 2

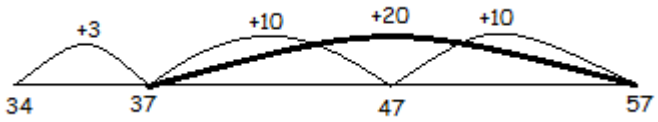
Solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures

Apply their increasing knowledge of mental and written methods.

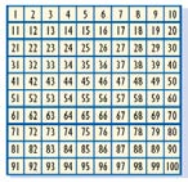
Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.

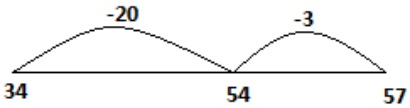
<table>
<tr><th>Addition</th><th>Subtraction</th></tr>
<tr><td colspan="2">Adding ten and multiples of ten from any number

 
15 + 10 = 25
15 + 30 = 45

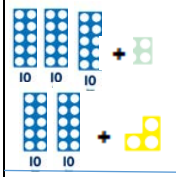
<u>Number line:</u>
34 + 23
 

Partition 23 = 20 + 3
Add ones first.
Then tens (leading to jumps of multiples of ten).
(See bridging larger numbers).</td><td>Subtracting ten and multiples of ten from any number

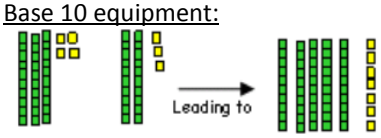
 
47 – 10 = 37
47 – 40 = 7
Using number lines, number squares and Numicon.

<u>Number Line:</u>
57 – 23
 

Partition 23 = 20 + 3
Subtract ones first.
Then tens (leading to jumps of multiples of ten).
(See bridging larger numbers).</td></tr>
<tr><td><u>Partitioning</u>
Using Numicon:

 
50 + 7 =57
<u>Partitioning</u>
Using arrow cards:
30 + 4 = 34
20 + 3 = 23
50 + 7 = 57

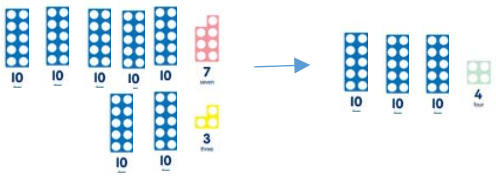
 
</td><td><u>Partitioning</u>
34 + 23 = 57

<u>Base 10 equipment:</u>
 

Children create the two sets with Base 10 equipment and then combine; ones with ones, tens with tens.
Beginning with the ones in preparation for formal columnar method.</td><td><u>Take away</u>
57 – 23 = 34

<u>Base 10 equipment:</u>

Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition.
 

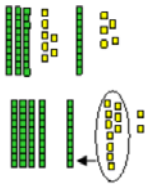
Using Numicon:
  </td></tr>
</table>

Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging. What happens if the ones are greater than 10? Introduce the term 'exchange'. Using the Base 10 equipment, children exchange ten ones for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

Base 10 equipment:

37 + 15 = 52

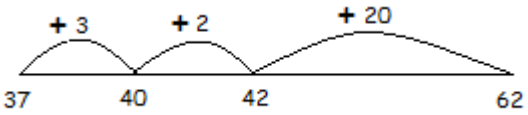


Discuss counting on from the larger number irrespective of the order of the calculation.

Number line

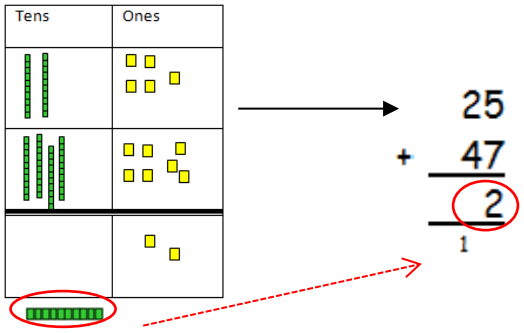
37+25 = 62

(37 + 3) + (2 + 20) = 62

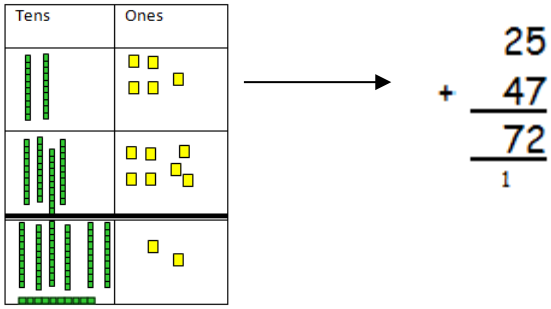


Compact method

Only when children can confidently calculate using the above methods do they move on to compact method.

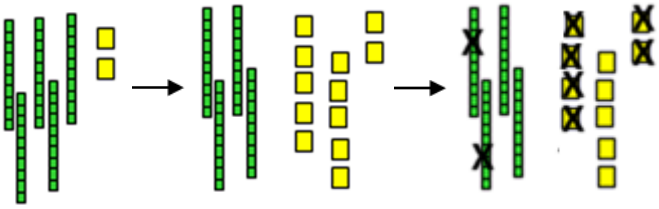


Leading to



Base 10 equipment:

52 – 37 = 15



Year 3

Add and subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens and a three-digit number and hundreds.

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.

Estimate the answer to a calculation and use inverse operations to check answers.

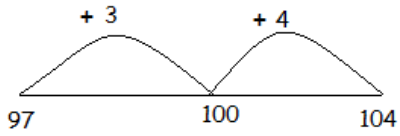
Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Addition	**Subtraction**
Once 2 digit addition is secure move onto 3 digits.	Once 2 digit subtraction is secure move onto 3 digits.

Number lines

To support mental methods and for teaching bridging over 100.

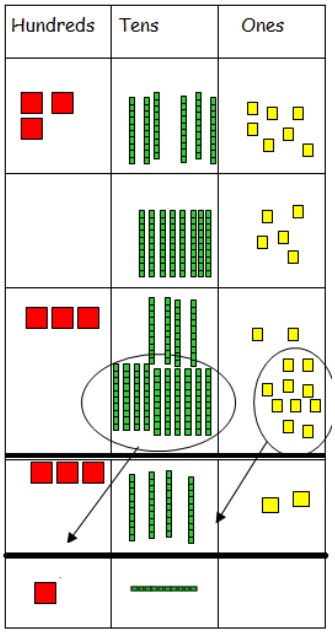
97 + 7 = 104



Partitioning using arrow cards:

Partitioning should be taught alongside use of base ten and formal written methods.

Compact method

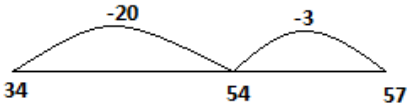


Leading to:

$$\begin{array}{r} 367 \\ +\ 85 \\ \hline 452 \\ 11 \end{array}$$

Number lines

57 – 23

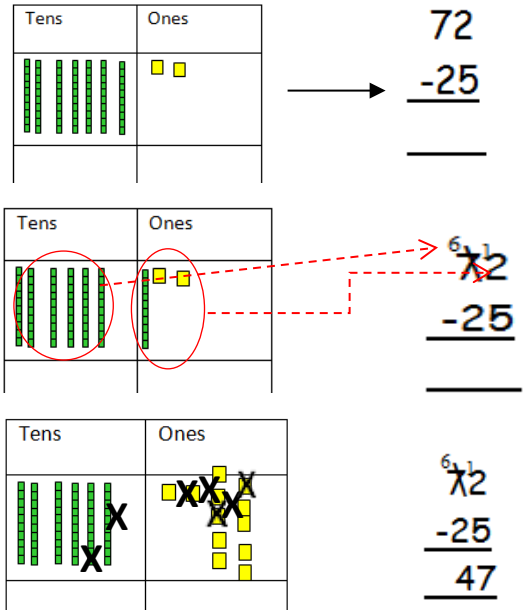


Partition 23 = 20 + 3

Subtract ones first

Then tens (leading to jumps of multiples of ten)

Compact decomposition



Ensure children can exchange:

ones e.g:

$$\begin{array}{r} 6\,\overset{1}{\not 7}\,\overset{1}{\not 2} \\ -\ 25 \\ \hline 47 \end{array}$$

tens e.g:

$$\begin{array}{r} \overset{1}{\not 1}36 \\ -\ 82 \\ \hline 54 \end{array}$$

ones and tens e.g:

$$\begin{array}{r} 2\,\overset{11}{\not 3}\,\overset{1}{\not 4} \\ -\ 126 \\ \hline 188 \end{array}$$

double exchange e.g:

$$\begin{array}{r} 2\,\overset{9}{\not 3}\,\overset{1}{\not 4} \\ -\ 126 \\ \hline 174 \end{array}$$

Year 4

Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.

Estimate and use inverse operations to check answers to a calculation.

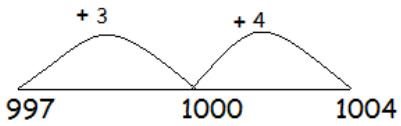
Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Addition	**Subtraction**
Once 3 digit formal written method is secure move onto 4 digits and 2 decimal places.	

Number line:

For mental methods and teaching bridging:

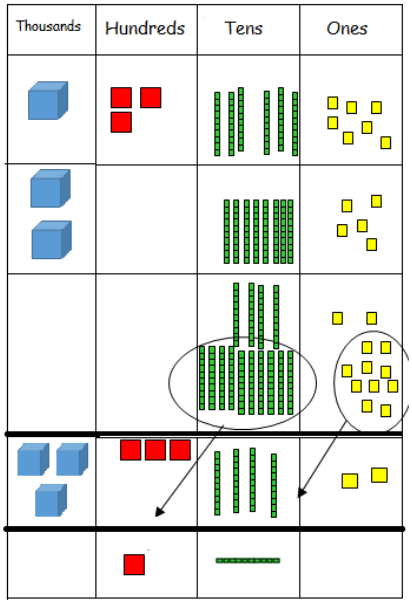
997 + 7 = 1004



Partitioning using arrow cards:

Partitioning should be taught alongside use of base ten and formal written methods.

Compact method



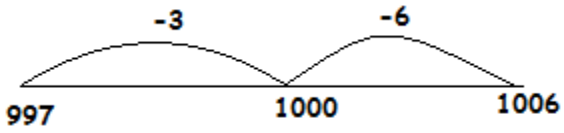
Leading to:

$$\begin{array}{r} 1{,}367 \\ +\ 2{,}085 \\ \hline 3{,}452 \\ \hline 11 \end{array}$$

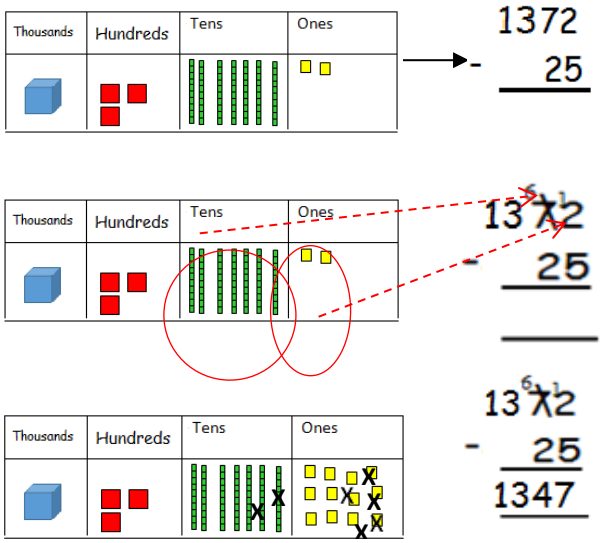
Number line

For mental methods and teaching bridging:

1006 – 9 = 997



Compact method



Ensure children can exchange:

ones e.g:

$$\begin{array}{r} 6\,\overset{1}{\not 7}\,\overset{1}{2} \\ -\ 25 \\ \hline 47 \end{array}$$

tens

hundreds

ones and tens

ones and hundred

tens and hundred

double exchange

triple exchange

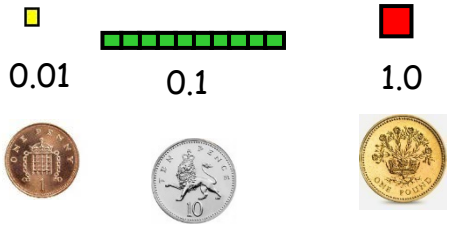
Decimals

Before adding and subtracting decimals ensure that children are confident in counting forwards and backwards in decimals – using counting stick, bead strings, money etc… to support.

Bead strings: 

Each bead represents 0.1, each different block of colour equal to 1.0.

Base 10 equipment and money:



Addition of decimals

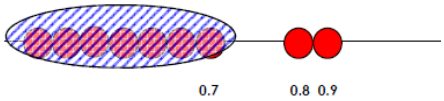
Counting both sets – starting at zero.

0.5 + 0.2 = 0.7

Leading to

Starting from the first set total, count on to the end of the second set.

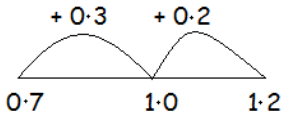
0.7 + 0.2 = 0.9



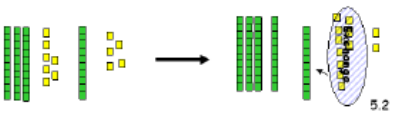
Bridging through 1.0

Encourage connections with number bonds.

0.7 + 0.5 = 1.2



Partitioning



3.7 + 1.5 = 5.2

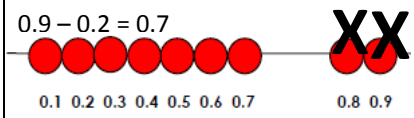
Leading to:

Compact method

Subtraction of decimals

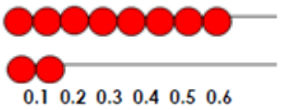
Take away model

0.9 – 0.2 = 0.7



Finding the difference:

0.8 – 0.2 =



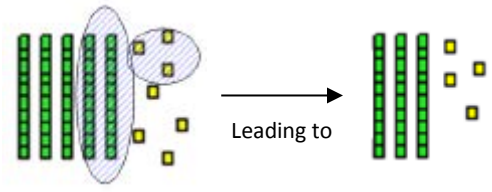
Bridging through 1.0

Encourage efficient partitioning.

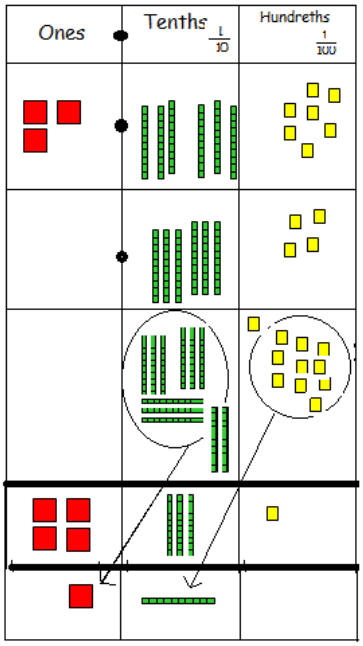
1.2 – 0.5 = 1.2 – 0.2 – 0.3 = 0.7

Partitioning

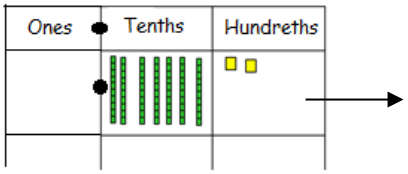
5.7 – 2.3 = 3.4



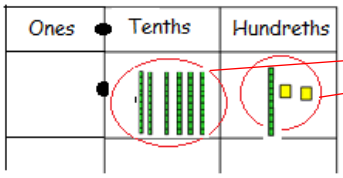
Compact method



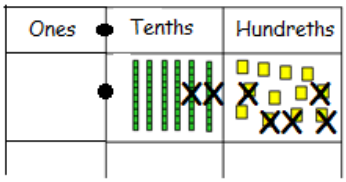
$$\begin{array}{r} 3{\cdot}67 \\ +\ 0{\cdot}64 \\ \hline 4{\cdot}21 \\ \hline 1\ 1 \end{array}$$



$$\begin{array}{r} 0{\cdot}72 \\ -\ 0{\cdot}25 \\ \hline \cdot \\ \hline \end{array}$$



$$\begin{array}{r} 0{\cdot}{}^{6}\not{7}{}^{1}2 \\ -\ 0{\cdot}\ 25 \\ \hline 0{\cdot} \\ \hline \end{array}$$



$$\begin{array}{r} 0{\cdot}{}^{6}\not{7}{}^{1}2 \\ -\ 0{\cdot}\ 25 \\ \hline 0{\cdot}\ 47 \\ \hline \end{array}$$

Year 5

Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction).

Add and subtract numbers mentally with increasingly large numbers.

Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.

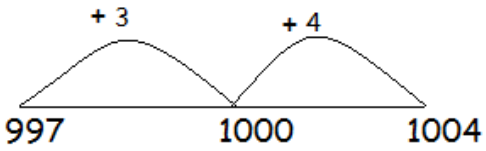
Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

<table>
<tr><th>Addition</th><th>Subtraction</th></tr>
<tr><td colspan="2">Once 4 digit formal written method is secure increase up to 7 digits and up to 3 decimal places.</td></tr>
<tr><td>

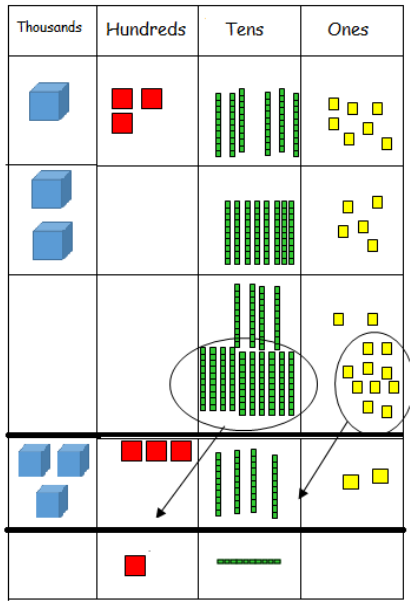
Number line:

For mental methods and teaching bridging:

997 + 7 = 1004



Compact method



Leading to:

1,367
+ 2,085
3,452
11

Progress to formal compact method with up to 7 digits and 3 decimal places:

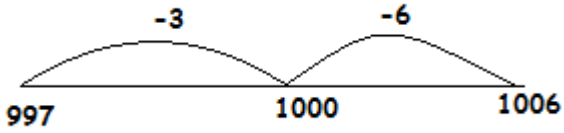
42,507·125
+ 38,408·496
80,915·621
1 1 11

</td><td>

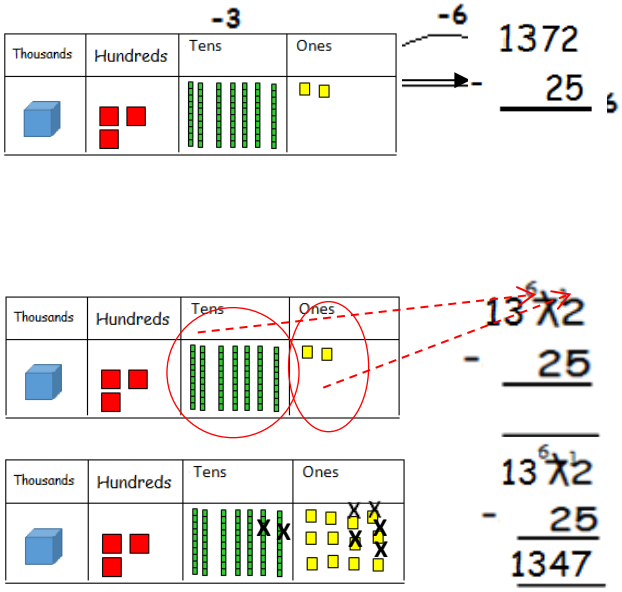
Number line

For mental methods and when numbers are close together or subtracting a small number:

1006 – 9 = 997

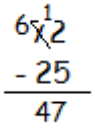


Compact method



Ensure children can exchange:

ones e.g



tens

hundreds

ones and tens

ones and hundreds

tens and hundreds

double exchange

triple exchange

etc…

</td></tr>
</table>

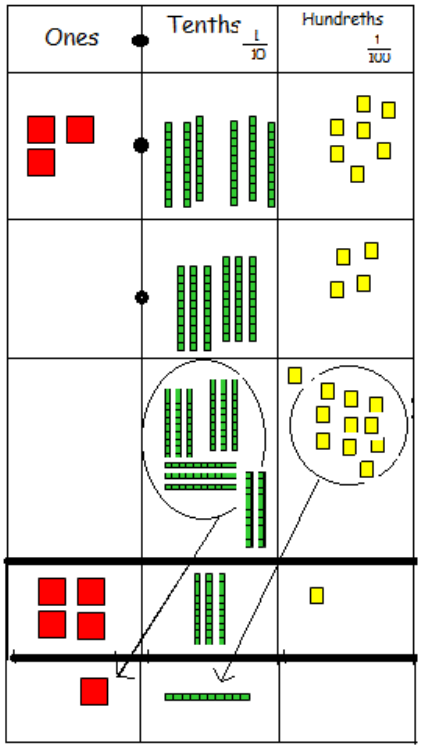
Decimals

Before adding and subtracting decimals ensure that children are confident in counting forwards and backwards in decimals – using counting stick, bead strings, money etc… to support.

Addition of decimals

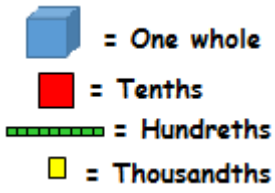
Compact method

For 2 decimal places:



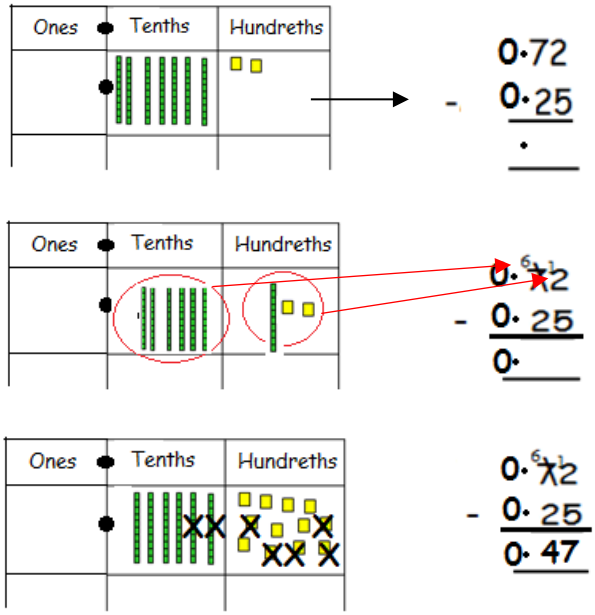
$$\begin{array}{r} 3{\cdot}67 \\ 0{\cdot}64 \\ + \quad 4{\cdot}31 \\ \hline 11 \end{array}$$

For three decimal places:

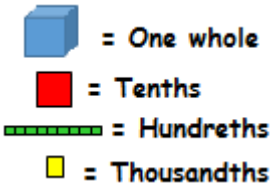


Subtraction of decimals

Compact method



For three decimal places:



Year 6

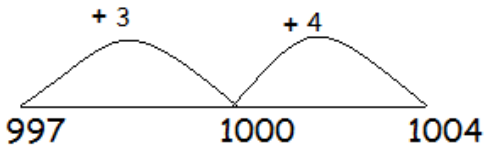
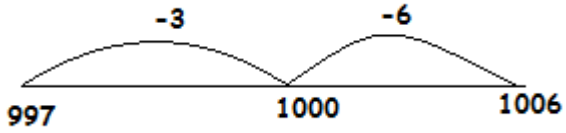
Perform mental calculations, including with mixed operations of addition and subtraction of large numbers.

Use their knowledge of the order of operations to carry out calculations involving addition and subtraction.

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Solve problems involving addition and subtraction.

Use estimation to check answers to calculations and determine, in the context of a problem an appropriate degree of accuracy.

Addition	**Subtraction**
Revision of formal written methods more than 4 digits and up to 3 decimal places.	
Number line: For mental methods and teaching bridging: 997 + 7 = 1004  **Compact method** 42,507·125 + 38,408·496 80,915·621 1 1 11	**Number line** For mental methods and when numbers are close together or subtracting a small number: 1006 – 9 = 997  **Compact method** 342,007·635 − 38,004·194 4,003·041

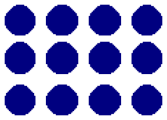
Progression in Multiplication and Division

Multiplication and division are connected.
Both express the relationship between a number of equal parts and the whole.

Part	Part	Part	Part
Whole			

The following array, consisting of four columns and three rows, could be used to represent the number sentences: -

3 x 4 = 12
4 x 3 =12
3 + 3 + 3 + 3 = 12
4 + 4 + 4 =12
And it is also a model for division
12 ÷ 4 = 3
12 ÷ 3 = 4
12 – 4 – 4 – 4 = 0
12 – 3 – 3 – 3 – 3 = 0



Gradation of difficulty (short multiplication)

1. TO x O no exchange.
2. TO x O extra digit in the answer.
3. TO x O with exchange of ones into tens.
4. HTO x O no exchange.
5. HTO x O with exchange of ones into tens.
6. HTO x O with exchange of tens into hundreds.
7. HTO x O with exchange of ones into tens and tens into hundreds.
8. As 4-7 but with greater number digits x O.
9. O.t x O no exchange.
10. O.t with exchange of tenths to ones.
11. As 9 - 10 but with greater number of digits which may include a range of decimal places x O.

Gradation of difficulty (short division)

1. TO ÷ O no exchange no remainder.
2. TO ÷ O no exchange with remainder.
3. TO ÷ O with exchange no remainder.
4. TO ÷ O with exchange, with remainder.
5. Zero in the quotient e.g. 816 ÷ 4 = **204.**
6. As 1-5 HTO ÷ O.
7. As 1-5 greater number of digits ÷ O.
8. As 1-5 with a decimal dividend e.g. 7.5 ÷ 5 or 0.12 ÷ 3.
9. Where the divisor is a two digit number.

See below for gradation of difficulty with remainders.

Dealing with remainders

Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly.

e.g.I have 62p. How many 8p sweets can I buy?

- Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed?

Gradation of difficulty for expressing remainders

1. Whole number remainder.
2. Remainder expressed as a fraction of the divisor.
3. Remainder expressed as a simplified fraction.
4. Remainder expressed as a decimal.

Key Vocabulary

Multiplication	Division
Multiply, times, product, groups of lots of, multiplied by, x times bigger.	Divide, division, divided by, share, sharing, equal groups of, equally, how many, remainder, quotient.
Consistent use of the vocabulary throughout maths sessions, in conjunction with visual models and practical experiences is essential to developing the concept of multiplication.	

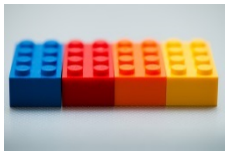
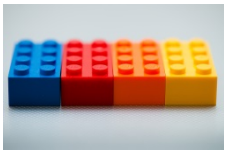
Foundation

By the end of the Foundation Stage, pupils are assessed against the following Early Learning Goals:

- They solve problems, including doubling, halving and sharing**.**

<table>
<tr><th colspan="2">Pupils will begin to record in the context of play or practical activities and problems.</th></tr>
<tr><th>Multiplication</th><th>Division</th></tr>
<tr><td>Solve problems including doubling

I have 2 pieces of lego how many would I have if I doubled the number of lego pieces? Encourage the children to physically find 4 more and then count how many they have. This can be done with various objects that are of interest to the children.

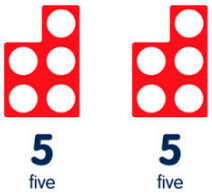
  

Using fingers is a good way to quickly work out doubles.

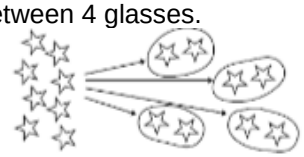
 
5 + 5 = 10

<u>Numicon</u>

Numicon is used to help visualise doubling.

 
5+5=10
Double 5 is 10</td><td>Solve problems, including doubling, halving and sharing.

Children will understand equal groups and share objects out in play and problem solving.

<u>Sharing equally</u>
6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.
 

 
</td></tr>
</table>

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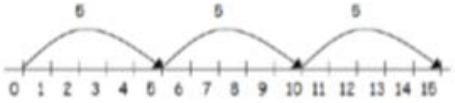
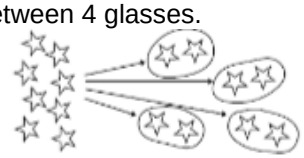
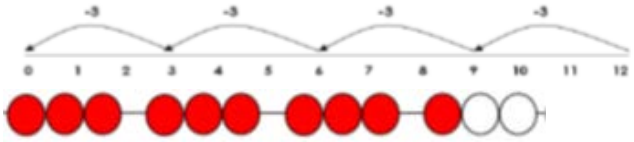
Year 1

Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

<table>
<tr><th>Multiplication</th><th>Division</th></tr>
<tr><td><u>Early experiences</u>
Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups.
  </td><td>Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.
  </td></tr>
<tr><td rowspan="2"><u>Repeated addition (repeated aggregation)</u>
3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3.
Children learn that repeated addition can be shown on a number line.
 
Children learn that repeated addition can be shown on a bead string.
 
Children also learn to partition totals into equal trains using Cuisenaire Rods.
  </td><td><u>Sharing equally</u>
6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.
  </td></tr>
<tr><td><u>Grouping or repeated subtraction</u>
There are 6 sweets. How many people can have 2 sweets each?
  </td></tr>
<tr><td></td><td><u>Repeated subtraction using a bead string or number line</u>
12 ÷ 3 = 4
 
The bead string helps children with interpreting division calculations, recognising that 12 ÷ 3 can be seen as ‘how many 3s make 12?’</td></tr>
</table>

Notes and guidance

Through grouping and sharing small quantities, pupils begin to understand multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities.
They make connections between arrays, number patterns, and counting in twos, fives and tens.

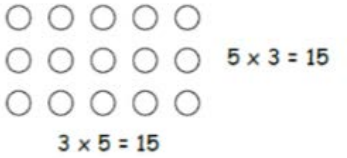
Year 2

Recall and use multiplications and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers. Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs. Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot. Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

<table>
<tr><th>Multiplication</th><th>Division</th></tr>
<tr><td>

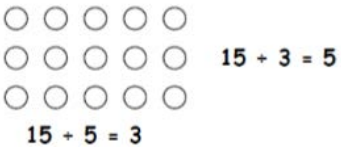
Arrays

Children learn to model a multiplication calculation using an array. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number.



</td><td>

Children learn to model a division calculation using an array.



</td></tr>
<tr><td>

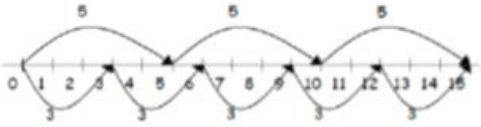
Commutativity

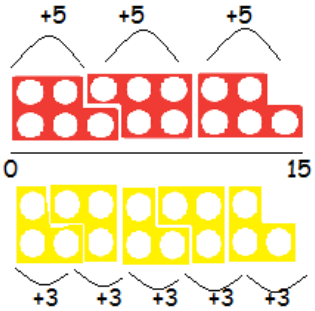
Children learn that 3 x 5 has the same total as 5 x 3.

This can also be shown on the number line.

3 x 5 = 15

5 x 3 = 15



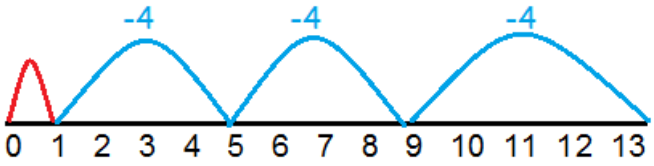


</td><td>

Grouping involving remainders

Children move onto calculations involving remainders.

13 ÷ 4 = 3 r1

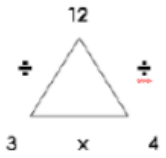


Or using Numicon.

</td></tr>
</table>

Inverse operations

Trios can be used to model the 4 related multiplication and division facts.



Children learn to state the 4 related facts.

3 x 4 = 12

4 x 3 = 12

12 ÷ 3 = 4

12 ÷ 4 = 3

Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

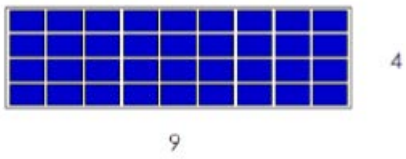
□x 5 = 20 3 x △ = 18 O x □= 32

24 ÷ 2 = □ 15 ÷O = 3 △ ÷10 = 8

Repeated addition for multiplication

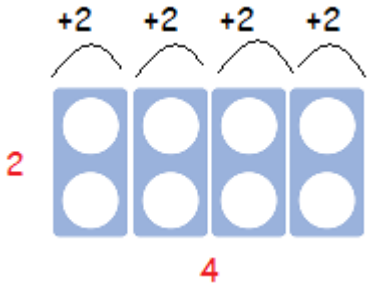
Arrays are also useful to help children visualise how to partition larger numbers into more useful representation.

9 x 4 = 36



Numicon

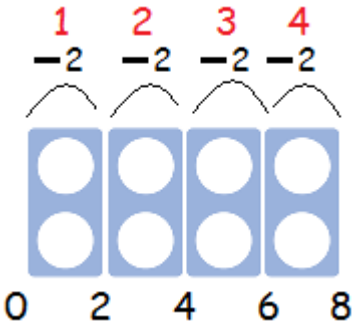
2 x 4 = 8



Repeated subtraction for division

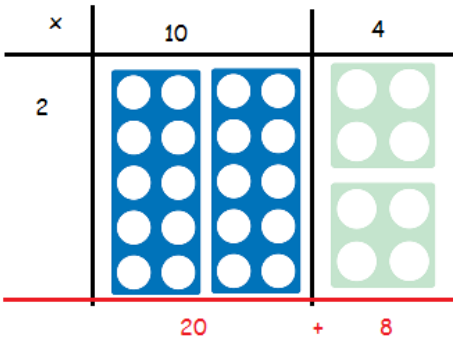
Children learn that division is **not** commutative and link this to subtraction.

8 ÷ 2 = 4



8 – 2 -2 -2 -2 = 0

Arrays leading into the grid method

Children continue to use arrays and partitioning, where appropriate, to prepare them for the grid method of multiplication. Arrays can be represented as 'grids' in a shorthand version and by using place value counters to show multiples of ten, hundred etc. **2 x 14 =**  **2 x 14 = 28**	

Notes and guidance

Pupils use a variety of language to describe multiplication and division. Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.

Pupils work with a range of resources. Numicon is most appropriate when multiplying and dividing with numbers up to 19, and dienes is more appropriate with numbers above 19. Pupils work with a range of contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities to arrays and repeated addition. They begin to relate these to fractions and measures (e.g. 40 ÷ 2 = 20, 20 is a half of 40).

They use commutativity and inverse relations to develop multiplicative reasoning (e.g. 4 × 5 = 20 and 20 ÷ 5 = 4).

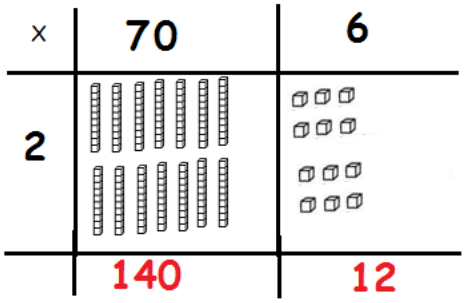
Year 3

Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Multiplication

Grid method

This written strategy is introduced for the multiplication of TO x O to begin with. It may require column addition methods to calculate the total.



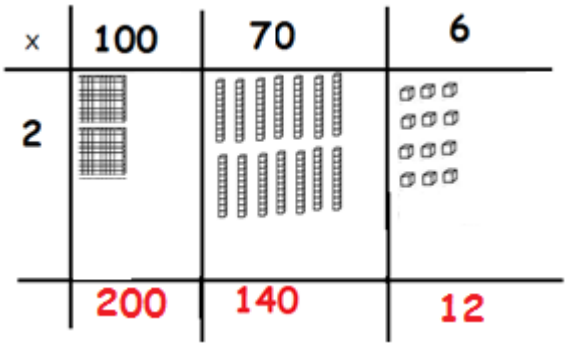
X	70	6
2	140	12

2 x 76 = 152

The idea of partitioning is used in this method to break the numbers down into more manageable chunks which are recombined in the final stages of the method.

Progress to HTO x O

2 x 176 =352



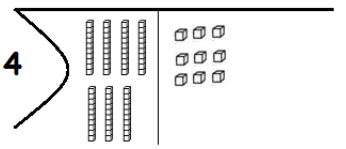
Division

Short division — dividing by a single digit

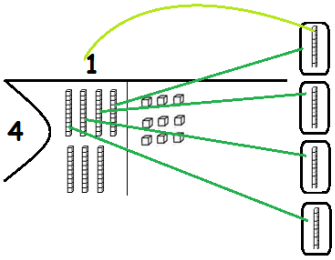
Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method. This model also connects division to **finding fractions** of discrete quantities.

We can begin to group counters into an array to show short division working.

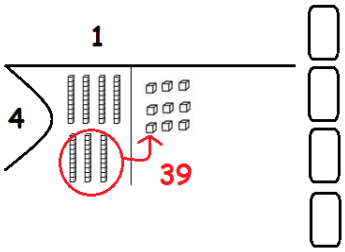
79 ÷ 4 =



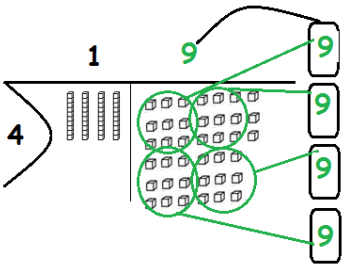
Partition 79 into tens and ones using dienes.



7 tens are shared equally into **4 groups** which is one ten each…



… 3 tens remaining. Carry into the ones column.



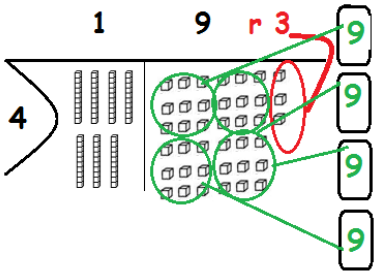
Exchange the **3 tens** for **30 ones.** There are now 39 ones. Repeat above: 39 ones are equally shared into **4 groups. 4 x 9 = 36**

Formal written method

Once the grid method has been consolidated progress on to vertical written multiplication:

23 x 7

	23 x 7	
multiply the ones	21	7 x 3
multiply the tens	140	7 x 20
add the totals	161	



There are 3 remaining.

79 ÷ 4 = 19 **r 3**

7 tens shared equally into **4 groups**. Which is **1 ten** with **3 tens remaining**.

1 9 r 3

4) 7 39

Exchange the **3 tens** for **30 ones** and the original 9 ones to make 39. How many 4s in 39? Put 9 in the ones column (4 x 9 = 36). There are 3 left remaining. Therefor e 79 ÷ 4 = 19 r 3

Notes and Guidance

Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.

Pupils develop efficient mental methods, for example, using commutativity and associativity (e.g. 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts (e.g. using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3).

Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.

Pupils solve simple problems in contexts, deciding which of the four operations to use and why. These include measuring and scaling contexts, (for example, ‘four times as high’, ‘eight times as long’, etc) and correspondence problems in which m objects are connected to n objects (e.g. 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

Year 4

Recall multiplication and division facts for multiplication tables up to 12 × 12.

Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers.

Recognise and use factor pairs and commutativity in mental calculations.

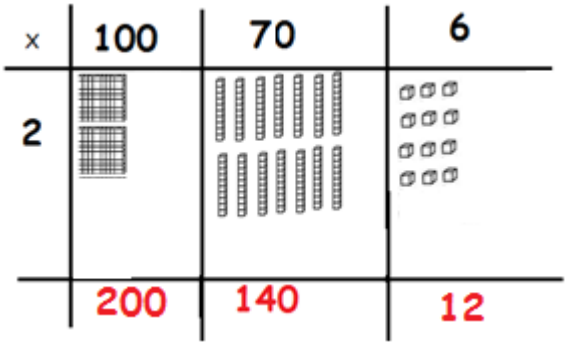
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Multiplication

Short multiplication — multiplying by a single digit

Ensure children consolidate their understanding of this formal written method using Dienes, see Year 3. **176 x 2 = 352**



4346 x 8 =

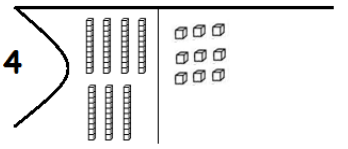
	4346 x 8	
multiply 8 by **6**	48	8 x 6
multiply 8 by **40**	320	8 x 40
multiply 8 by **300**	2400	8 x 300
multiply 8 by **4000**	32000	8 x 4000
total the numbers	34768	

Division

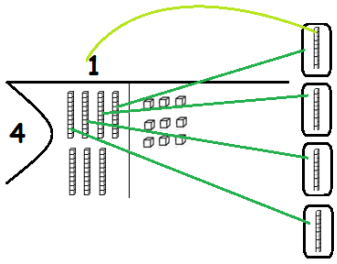
Short division — dividing by a single digit

Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method. This model also connects division to **finding fractions** of discrete quantities.

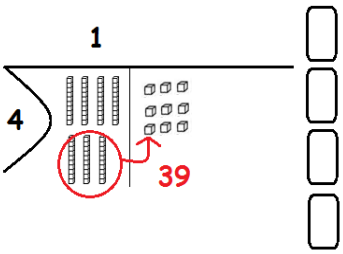
79 ÷ 4 =



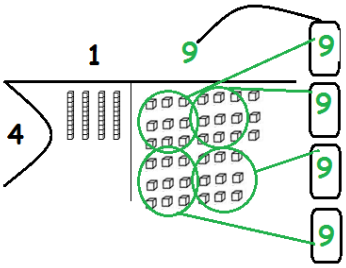
Partition 79 into tens and ones using dienes.



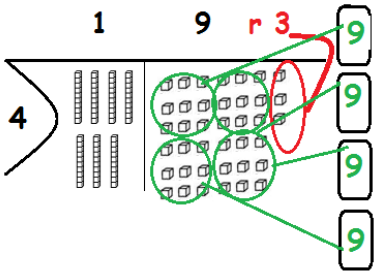
7 tens are shared equally into **4 groups** which is one ten each…



… 3 tens remaining. Carry into the ones column.



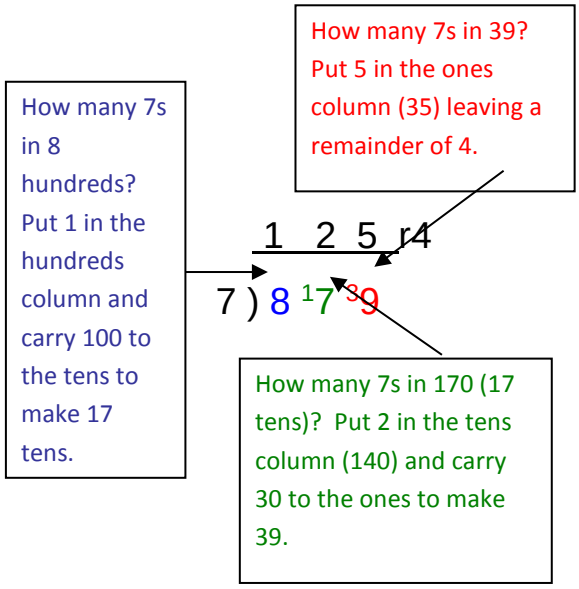
Exchange the **3 tens** for **30 ones.** There are now 39 ones. Repeat above: 39 ones are equally shared into **4 groups. 4 x 9 = 36.**



There are 3 remaining.

79 ÷ 4 = 19 **r 3**

This short written method should be taught alongside using dienes. Ensure children have consolidated their understanding of place before solely using a written method without apparatus.



Notes and Guidance

Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency. Pupils practise mental methods and extend this to three-digit numbers to derive facts (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6).

Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers. (see introduction).

Pupils write statements about the equality of expressions (e.g. use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations e.g. 2 x 6 x 5 = 10 x 6.

Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or three cakes shared equally between 10 children.

Year 5

Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers. Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers. Establish whether a number up to 100 is prime and recall prime numbers up to 19. Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers. Multiply and divide numbers mentally drawing upon known facts. Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context. Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000. Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}). Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes. Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign. Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Multiplication

Short and long multiplication

Consolidate understanding of multiplying up to 3digits by a single digit.

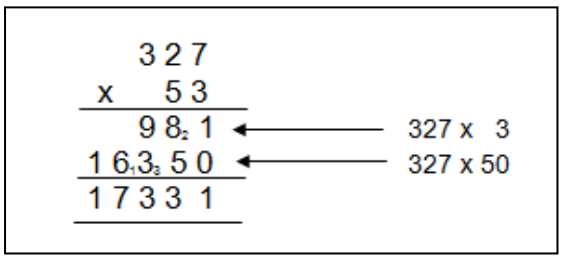
Once children are ready introduce compact method (also known as long multiplication) for HTU x TU, TU x TU.

e.g. 72 x 38

	72	(70 and 2)
	x 38	(30 and 8)
multiply unit by ones	16	8 x2
multiply unit by tens	560	8 x 70
multiply tens by ones	60	30 x 2
multiply tens by tens	2100	30 x 70
total the numbers	2736	

(Requires additional jottings. More efficient to use expanded or grid method.)

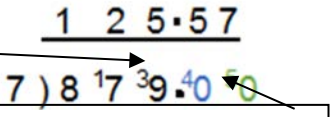
Once the expanded long multiplication is mastered, progress on to a further compact method:



Division

Short division

7 x 5 = 35. Therefore 4 ones are remaining. These are carried into the tenths column. Share 4.0 equally between 7. (Place value think 40) 7 x 5 = 35. Therefore place 5 in the tenths column.



7 x 5 = 35. Carry the remaining 5 across to the hundredths column. Share 5.0 equally between 7 (place value think 50) 7 x 7 = 49. Therefore place 7 in the hundredths column. Stop at two decimal places.

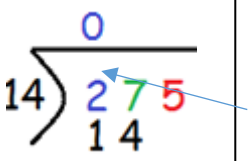
Up to 2 decimal places

Long Division

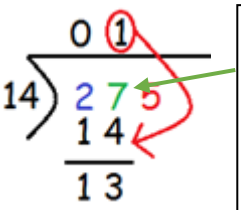
275 ÷ 14

First create a list of the 14 times tables:

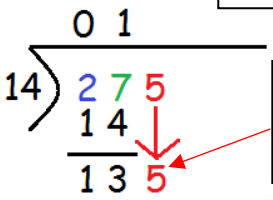
14 x 1 = 14
14 x 2 = 28
14 x 3 = 42
14 x 4 = 56
14 x 5 = 70
14 x 6 = 84
14 x 7 = 98
14 x 8 = 112
14 x 9 = 126
14 x 10 = 140



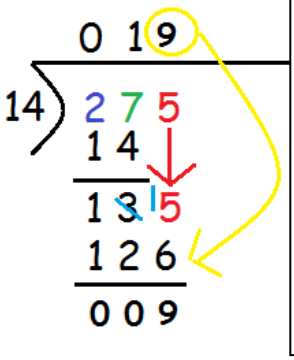
When we share the hundred blocks equally into 14 groups, how many in each group? As this equals 0, place 0 in the hundreds column and move onto the tens.



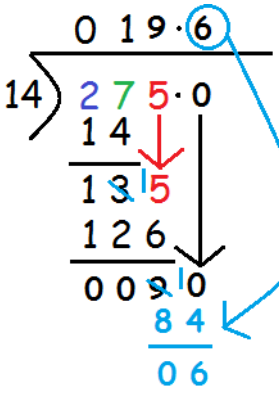
When we share 27 tens into 14 groups, how many in each group? How many are remaining? 1 group of 14 fits into 27 so therefore put 1 in the tens column. As 1 x 14 =14 place 14 below 27 and use subtraction to find how many tens are remaining?



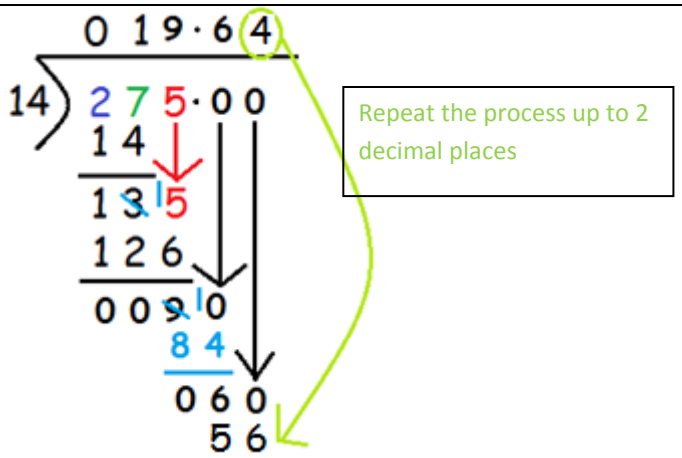
27 tens subtract 14 tens = 13 tens. Now bring down the 5 ones to make 135 ones.



When we share 135 ones into 14 groups, how many are in each group? How many are remaining? Refer to times tables list at start. Put 9 in the ones column (because 9 x 14 = 126) and place the answer 126 below 135 to use subtraction to find how many are remaining.



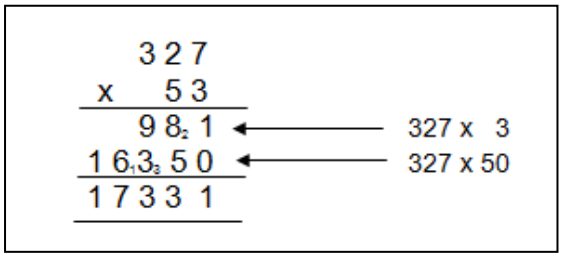
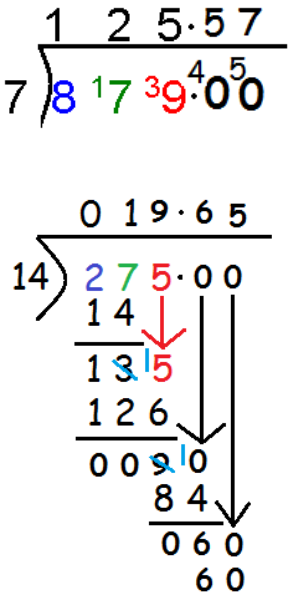
Insert decimal point. Bring the 0 down to make 90 (ignore place value of decimals at this stage). Share 90 equally into 14 groups. 14 x 6 = 84 so put 6 in the tenths column and place 84 (the answer) below 90 to begin subtraction.

Pupils practise and extend their use of the formal written methods of short multiplication and short division (see introduction). They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations. They use and understand the terms factor, multiple and prime, square and cube numbers. Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. 98 ÷ 4 = 98/4 = 24 r 2 = 24½ = 24.5 ≈ 25). Pupils use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres. Distributivity can be expressed as a(b + c) = ab + ac in preparation for using algebra. They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, 4 x 35 = 2 x 2 x 35; 3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10). Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example, 13 + 24 = 12 + 25; 33 = 5 x ?).

Year 6

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication. Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Divide numbers up to 4 digits by a two-digit whole number using the formal written method of short division where appropriate interpreting remainders according to the context. Perform mental calculations, including with mixed operations and large numbers. Identify common factors, common multiples and prime numbers. Use their knowledge of the order of operations to carry out calculations involving the four operations. Solve problems involving addition, subtraction, multiplication and division. Use estimation to check answers to calculations and determine, in the context of a problem an appropriate degree of accuracy.

Multiplication	**Division**
Short and long multiplication Consolidate understanding of short and long multiplication up to 2 decimals.  	**Short and long division** Consolidate understanding of short and long division up to 2 decimal places. 

Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see introduction). They undertake mental calculations with increasingly large numbers and more complex calculations. Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency. Pupils round answers to a specified degree of accuracy, e.g. to the nearest 10, 20, 50 etc, but not to a specified number of significant figures. Pupils explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9. Common factors can be related to finding equivalent fractions.